LECTURE NO 27

Magnetostatics

Topics

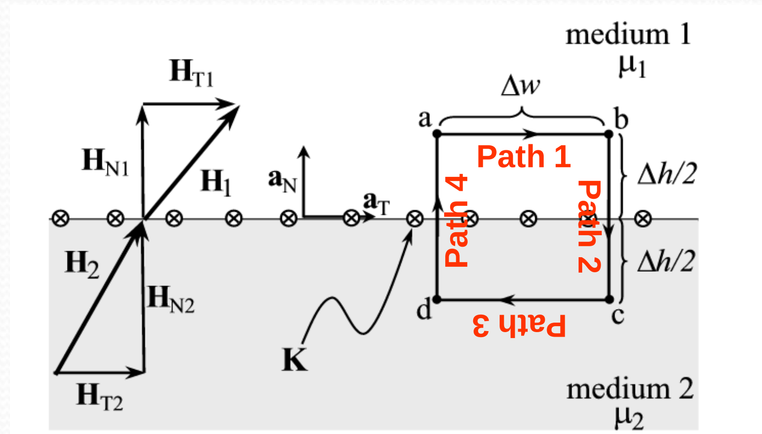
magnetic boundary conditions,

inductors and inductances,

magnetic energy

Magnetostatic Boundary Conditions

Will use Ampere's circuital law and Gauss's law to derive normal and tangential boundary conditions for magnetostatics.



Ampere's circuit law:

$$\oint \mathbf{H} \cdot d\mathbf{L} = I_{enc}$$

The current enclosed by the path is

$$I_{enc} = \int K dW = K \Delta w.$$

We can break up the circulation of **H** into four integrals:

$$\oint \mathbf{H} \cdot d\mathbf{L} = \int_a^b + \int_b^c + \int_c^d + \int_d^a (\mathbf{H} \cdot d\mathbf{L}) = K \Delta w.$$

Path 1: $$\int_a^b \mathbf{H} \cdot d\mathbf{L} = \int_0^{\Delta w} H_{T1} \mathbf{a}_T \cdot dL \mathbf{a}_T = H_{T1} \Delta w.$$

Path 2: $$\int_b^c \mathbf{H} \cdot d\mathbf{L} = \int_{\Delta h/2}^{0} H_{N1} \mathbf{a}_N \cdot dL \mathbf{a}_N + \int_0^{-\Delta h/2} H_{N2} \mathbf{a}_N \cdot dL \mathbf{a}_N = -(H_{N1} + H_{N2}) \frac{\Delta h}{2}$$

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Path 3: $$\int_c^d \mathbf{H}\cdot d\mathbf{L} = \int_{\Delta w}^{0} H_{\mathrm{T}2}\mathbf{a}_{\mathrm{T}}\cdot dL\mathbf{a}_{\mathrm{T}} = -H_{\mathrm{T}2}\Delta w.$$

Path 4: $$\int_d^a \mathbf{H}\cdot d\mathbf{L} = \int_{-\Delta h/2}^{0} H_{\mathrm{N}2}\mathbf{a}_{\mathrm{N}}\cdot dL\mathbf{a}_{\mathrm{N}} + \int_{0}^{\Delta h/2} H_{\mathrm{N}1}\mathbf{a}_{\mathrm{N}}\cdot dL\mathbf{a}_{\mathrm{N}} = \left(H_{\mathrm{N}1}+H_{\mathrm{N}2}\right)\frac{\Delta h}{2}$$

Now combining our results (i.e., Path 1 + Path 2 + Path 3 + Path 4), we obtain

ACL: $$\oint \mathbf{H}\cdot d\mathbf{L} = I_{enc}$$

$$\oint \mathbf{H}\cdot d\mathbf{L} = \left(H_{\mathrm{T}1}-H_{\mathrm{T}2}\right)\Delta w \quad \Longleftrightarrow \quad I_{enc} = \int K dW = K\Delta w$$

Equating

Tangential BC: $$\boxed{H_{\mathrm{T}1}-H_{\mathrm{T}2}=K}$$

A more general expression for the first magnetostatic boundary condition can be written as

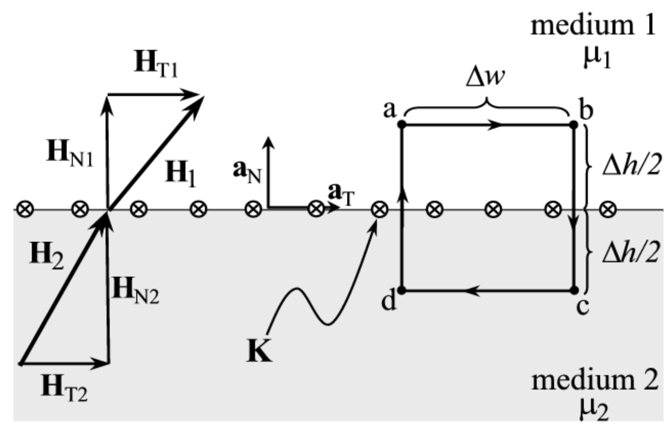
$$\boxed{\mathbf{a}_{21}\times\left(\mathbf{H}_1-\mathbf{H}_2\right)=\mathbf{K}}$$

where $\mathbf{a}_{21}$ is a unit vector normal going from media 2 to media 1.

Magnetostatic Boundary Conditions

Special Case: If the surface current density K = 0, we get

$$H_{T1} - H_{T2} = K \xrightarrow{\text{If } K = 0} H_{T1} = H_{T2}$$



The tangential magnetic field intensity is continuous across the boundary when the surface current density is zero.

Important Note:

We know that $\mathbf{B} = \mu_o \mu_r \mathbf{H}$ (or) $\mathbf{H} = \dfrac{\mathbf{B}}{\mu_o \mu_r}$

Using the above relation, we obtain

$$H_{T1} = H_{T2} \longrightarrow \frac{B_{T1}}{\mu_o \mu_1} = \frac{B_{T2}}{\mu_o \mu_2}$$

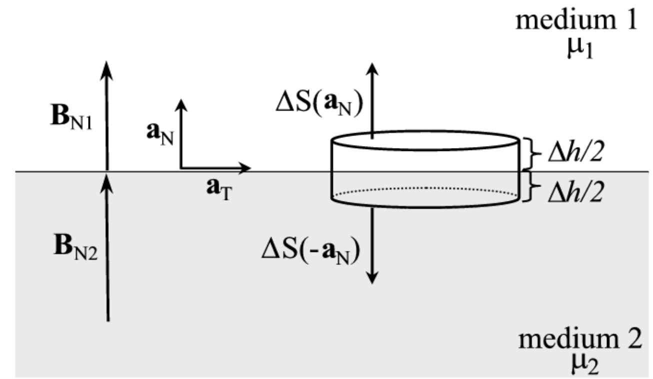
Therefore, we can say that $B_{T1} \neq B_{T2}$

The tangential component of the magnetic flux density B is not continuous across the boundary.

Magnetostatic Boundary Conditions

Gauss's Law for Magnetostatic fields:

$$\oint \mathbf{B} \cdot d\mathbf{S} = 0$$



To find the second boundary condition, we center a Gaussian pillbox across the interface as shown in Figure.

We can shrink Δh such that the flux out of the side of the pillbox is negligible. Then we have

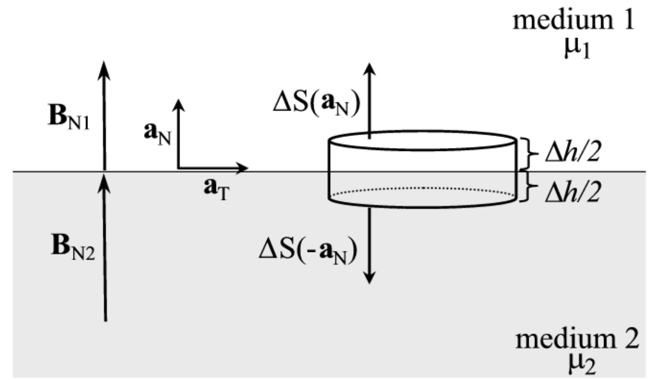
$$\oint \mathbf{B} \cdot d\mathbf{S} = \int B_{N1}\mathbf{a}_N \cdot dS\mathbf{a}_N + \int B_{N2}\mathbf{a}_N \cdot dS(-\mathbf{a}_N)$$

$$= \left(B_{N1} - B_{N2}\right)\Delta S = 0.$$

Normal BC: $\boxed{B_{N1} = B_{N2}}.$

Magnetostatic Boundary Conditions

Normal BC: $$B_{N1} = B_{N2}$$



Thus, we see that the normal component of the magnetic flux density must be continuous across the boundary.

Important Note:

We know that $\mathbf{B} = \mu_o \mu_r \mathbf{H}$

Using the above relation, we obtain

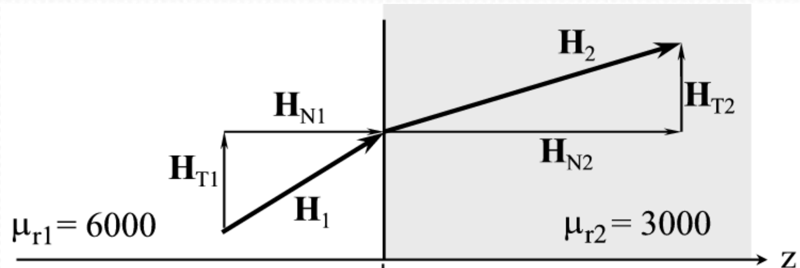
$$B_{N1} = B_{N2} \quad \Rightarrow \quad \mu_o \mu_1 H_{N1} = \mu_o \mu_2 H_{N2}$$

Therefore, we can say that $H_{N1} \neq H_{N2}$

The normal component of the magnetic field intensity is not continuous across the boundary (but the magnetic flux density is continuous).

Magnetostatic Boundary Conditions

<u>Example 3.11</u>: The magnetic field intensity is given as $\mathbf{H}_1 = 6\mathbf{a}_x + 2\mathbf{a}_y + 3\mathbf{a}_z$ (A/m) in a medium with $\mu_{r1} = 6000$ that exists for $z < 0$. We want to find $\mathbf{H}_2$ in a medium with $\mu_{r2} = 3000$ for $z > 0$.



$\mathbf{H}_1 = 6\mathbf{a}_x + 2\mathbf{a}_y + 3\mathbf{a}_z \; A/m$	(g) $\mathbf{H}_2 = 6\mathbf{a}_x + 2\mathbf{a}_y + 6\mathbf{a}_z \; A/m$
(a) $\mathbf{H}_{N1} = 3\mathbf{a}_z$	(f) $\mathbf{H}_{N2} = \mathbf{B}_{N2}/\mu_{r2}\mu_o = 6\mathbf{a}_z$
(b) $\mathbf{H}_{T1} = 6\mathbf{a}_x + 2\mathbf{a}_y$	(c) $\mathbf{H}_{T2} = \mathbf{H}_{T1} = 6\mathbf{a}_x + 2\mathbf{a}_y$
(d) $\mathbf{B}_{N1} = \mu_{r1}\mu_o\mathbf{H}_{N1} = 18{,}000\mathbf{a}_z$	(e) $\mathbf{B}_{N2} = \mathbf{B}_{N1} = 18{,}000\mathbf{a}_z$

Step (a) and (b): The first step is to break $\mathbf{H}_1$ into its normal component (a) and its tangential component (b).

Step (c): With no current at the interface, the tangential component is the same on both sides of the boundary.

Step (d): Next, we find $\mathbf{B}_{N1}$ by multiplying $\mathbf{H}_{N1}$ by the permeability in medium 1.

Step (e): This normal component **B** is the same on both sides of the boundary.

Step (f): Then we can find $\mathbf{H}_{N2}$ by dividing $\mathbf{B}_{N2}$ by the permeability of medium 2.

Step (g): The last step is to sum the fields .